Algebra I
Unit 7: Exponents, Exponential Functions, and Nonlinear Graphs

Time Frame: Approximately five weeks

Unit Description

This unit is an introduction to exponential functions and their graphs. Special emphasis is given to examining their rate of change relative to that of linear equations. Distinguishing between linear and exponential functions based on function statements, tables, and graphs is also an integral part of this unit. Focus is on the real-life applications of exponential growth and decay. Laws of exponents are introduced as well as the simplification of polynomial expressions. Beginning in 2013, analyzing quadratic functions will be included in this unit. Determining the solutions of quadratics by finding square roots, factoring, completing the square, and applying the quadratic formula also will be included. Graphing and analyzing quadratic functions as compared to graphing and analyzing linear and exponential functions will be included as well.

Student Understandings

Students develop the understanding of exponential growth and its relationship to repeated multiplications, rather than repeated additions, and its relationship to exponents and radicals. Students should be able to simplify expressions containing fractional exponents. Students recognize, graph, and write symbolic representations for simple exponential relationships of the form $a \cdot b^x$. They are able to evaluate and describe exponential changes in a sequence by citing the rules involved. Students are able to add, subtract, and multiply polynomial expressions. In addition, students should be able to solve quadratic equations using graphing, square roots, factoring, completing the square, and the quadratic formula.

Guiding Questions

1. Can students recognize the presence of an exponential rate of change from data, equations, or graphs?
2. Can students develop an expression or equation to represent a straightforward exponential relation of the form $y = a \cdot b^x$?
3. Can students differentiate between the rates of growth for exponential and linear relationships?
4. Can students use exponential growth and decay to model real-world relationships?
5. Can students use laws of exponents to simplify polynomial expressions, including those expressions containing fractional exponents?
6. Can students add, subtract, and multiply polynomials, recognizing that such operations are under a closed system?

7. Can students factor quadratic functions to determine solutions?
8. Can students solve quadratic equations using various methods, such as graphing, completing the square, and the quadratic formula?
9. Can students differentiate among linear, exponential, and quadratic functions?

Unit 7 Grade-Level Expectations (GLEs) and Common Core State Standards (CCSS)

Grade-Level Expectations	
GLE #	**GLE Text and Benchmarks**
Number and Number Relations	
2.	Evaluate and write numerical expressions involving integer exponents (N-2-H)
GLE #	**GLE Text and Benchmarks**
Algebra	
8.	Use order of operations to simplify or rewrite variable expressions (A-1-H) (A-2-H)
11.	Use equivalent forms of equations and inequalities to solve real-life problems (A-1-H)
15.	Translate among tabular, graphical, and algebraic representations of functions and real-life situations (A-3-H) (P-1-H) (P-2-H)
Data Analysis, Probability, and Discrete Math	
29.	Create a scatter plot from a set of data and determine if the relationship is linear or nonlinear (D-1-H) (D-6-H) (D-7-H)
Patterns, Relations, and Functions	
36.	Identify the domain and range of functions (P-1-H)
CCSS for Mathematical Content	
CCSS#	**CCSS Text**
The Real Number System	
N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*
Seeing Structure in Expressions	
A-SSE.1	Interpret expressions that represent a quantity in terms of its context
Arithmetic with Polynomials and Rational Expressions	
A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
Reasoning with Equations and Inequalities	
A-REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

	b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers *a* and *b*.
A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Linear, Quadratic, and Exponential Models	
F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
F-LE.5	Interpret the parameters in a linear, quadratic, or exponential function in terms of a context.
Interpreting Categorical and Quantitative Data	
S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

Sample Activities

Activity 1: Evaluation of Linear and Exponential Functions (GLEs: 2, 15, 29, 36; CCSS: A-SSE.1, F-LE.1, F-LE.2, F-LE.5, S-ID.6)

Materials List: paper, pencil, Vocabulary Self-Awareness Chart BLM, Evaluation: Linear and Exponential Functions BLM, Graphic Organizer BLM, graphing calculator

Preview the unit by having students complete the *vocabulary self-awareness chart* (view literacy strategy descriptions). *Vocabulary self-awareness charts* have been used often to build technical vocabulary fluency throughout the curriculum. A BLM is provided for students to use as they begin to study exponential and other nonlinear functions. As students become proficient with the vocabulary, select student-made *vocabulary self-awareness charts* to place on the bulletin board to remind students about linear and exponential functions. *Vocabulary self-awareness charts* may also be used to review for tests and quizzes.

Have students use the Evaluation: Linear and Exponential Functions BLM to complete this activity. The BLM gives students the two functions, $f(x) = 3x$ and $f(x) = 3^x$. Have students generate an input-output table using the same domain for both functions. Have students plot the ordered pairs for each function and connect them. Next, have students calculate the difference between successive *y*-coordinates in each function and compare them. Discuss with students the fact that the rate of change varies for a nonlinear function as opposed to the constant rate of change found in linear functions. (This is called the method of finite differences which will be studied in depth in Algebra II.) Relate this varying rate of change to the shape of the graph and the type of function. Have students complete the BLM. Conduct a class discussion on what happens to the graph when the base, *b*, changes in the function $y = b^x$. Discuss with students the difference between the exponential growth function and the exponential decay function. Also, remind students of the meaning of the negative exponent, that is, that a negative exponent requires that the reciprocal of the base be used (example: $3^{-2} = \frac{1}{9}$). Therefore, $b^{-x} = \left(\frac{1}{b}\right)^x$.

Have students use a *graphic organizer* (view literacy strategy descriptions) to compare and contrast a linear function and an exponential function. A *graphic organizer* is an instructional tool that allows students to give a pictorial representation of a topic. Provide students with the Graphic Organizer BLM of a blank compare and contrast diagram. Have students write a definition of each type of function listed on the BLM. Have students list the characteristics which make the functions different on the correct side of the *graphic organizer*, then have students list the characteristics that the functions have in common in the middle of the diagram.

Provide students with examples of real-life exponential functions, such as the one shown below and lead them in a class discussion of the characteristics of the function.

Example:
Atoms of radioactive elements break down very slowly into atoms of other elements. The amount of a radioactive element remaining after a given amount of time is an exponential relationship. Given an 80-gram sample of an isotope of mercury, the number of grams (*y*) remaining after *x* days can be represented by the formula $y = 80\left(0.5^x\right)$.

- Create a table for this function to show the number of grams remaining for 0, 1, 2, 3, 4, 5, 6, and 7 days. Identify the dependent and independent variables.

0	*80*
1	*40*
2	*20*
3	*10*
4	*5*
5	*2.5*
6	*1.25*
7	*.625*

- If half-life is defined as the time it takes for half the atoms to disintegrate, what is the half-life of this isotope? *(1 day)*
- Use a graphing calculator to display the graph.

Activity 2: The King's Chessboard – Modeling exponential growth (GLEs: 15, <u>29</u>; CCSS: A-SSE.1, F-LE.1, F-LE.2, F-LE.5, S-ID.6)

Materials List: paper, pencil, graph paper, rice, Chessboard BLM**,** graphing calculator (optional)

This activity has not changed because it already incorporates the CCSS.

Present students with the following folktale from India (The children's book *The King's Chessboard* by David Birch could also be used to set the activity.):

> A man named Sissa Ben Dahir invented the game of chess. The king liked the game so much that he wanted to reward Sissa with 64 gold pieces, one for each square on the chessboard. Instead, Sissa asked for 1 grain of wheat for the first square on the chessboard, 2 grains for the second, 4 grains for the third, 8 grains for the fourth, etc.
> How many grains of wheat will Sissa receive for the 64^{th} square? (2^{63})

Have groups of three students model the problem using grains of rice and the Chessboard BLM. Have them construct a table for the square's number and the number of grains of wheat, then graph the data on graph paper. The graphing calculator can also be used to graph a scatter plot. Have students write the exponential equation that models the situation and answer the question in the problem.

Revisit the paper folding activity and the Pay Day activity from Unit 1, and have students compare and contrast the two activities and their demonstration of exponential growth.

Activity 3: What's with my M&Ms®? Modeling exponential decay (GLEs: 15, <u>29</u>; CCSS: A-SSE.1, F-LE.1, F-LE.2, F-LE.5, S-ID.6)

Materials List: paper, pencil, Radioactive M&Ms® BLM, M&Ms®, ziploc bags, paper plates, graphing calculator, graph paper

This activity has not changed because it already incorporates the CCSS.

Have students use the Radioactive M&Ms® BLM to complete this activity. Give each student a ziploc bag with 50 M&Ms**®**. Have them follow the directions on the Radioactive M&Ms® BLM to collect their data**.** Have students graph the data by hand and with the graphing calculator. Students should then predict the equation based on the form of exponential decay. Have them use the calculator to find the equation for the exponential regression. Discuss with students exponential decay and the significance of the values of *a* and *b* in the exponential regression.

Revisit the paper folding activity in Unit 1 and compare and contrast the two examples of exponential decay.

Activity 4: Vampire simulation (GLEs: 11, 15, <u>29</u>; CCSS: A-SSE.1, F-LE.1, F-LE.2, F-LE.5, S-ID.6)

Materials List: paper, pencil, graph paper, graphing calculator (optional)

This activity has not changed because it already incorporates the CCSS.

Explore the common vampire folklore with students: When a vampire bites another person, that person becomes a vampire. If three vampires come into town and each vampire bites another person each hour, how long will it take for the entire town to become vampires?

Have one student at the board make a table of the following experiment using hour as the independent variable and the number of vampires as the dependent variable. Begin with three students (vampires) in front of the classroom. Have each student pick (bite) another student to bring in front of the classroom. Now there are six vampires. Have those six students each bring a student to the front of the classroom. Continue until all of the students have become vampires. Have the students return to their desks and copy the table, graph the data by hand, and find the equation to model the situation. Discuss with students the development of the equation of the form $y = a \cdot b^x$ ($y = 3 \cdot 2^x$), where x = the number of hours that have passed and y = the total number of vampires after each selection process. They should then use the equation to predict how long it would take for the entire town to become vampires. Students can use the graphing calculator to check their answers.

Activity 5: Exponential Decay in Medicine (GLEs: 11, 15, <u>29</u>; CCSS: A-SSE.1, F-LE.1, F-LE.2, F-LE.5, S-ID.6)

Materials List: paper, pencil, clear glass bowls, measuring cups, water, food coloring, graph paper, graphing calculator (optional)

This activity has not changed because it already incorporates the CCSS.

Pose the following problem:

> In medicine, it is important for doctors to know how long medications are present in a person's bloodstream. For example, if a person is given 300 mg of a pain medication, and every four hours the kidneys eliminate 25% of the drug from the bloodstream, is it safe to give another dose after four hours? When will the drug be completely eliminated from the body?

The following activity could be done in groups or conducted as a demonstration by the teacher. Students will need clear glass bowls, a measuring cup, 4 cups of water, 5 drops of food coloring. Have students pour 4 cups of water into the bowl and add the food coloring to it. Have students simulate the elimination of 25% of the drug by removing one cup of the colored water and adding one cup of clear water to the bowl. Have students repeat the steps and investigate how many times the steps need to be repeated until the water is clear. Have students make a table of values using end-of-time period (every four hours) as the independent variable and amount of medicine left in the body as the dependent variable. Help students develop the equation to model the situation ($y = 300 \cdot 0.75^x$). Have them graph the equation by hand or with the graphing calculator to investigate when the medicine will be completely eliminated from the body. Question students about whether the function will ever reach zero.

Activity 6: Exploring Laws of Exponents (GLEs: 2, 8; CCSS: N-RN.1, A-APR.1)

Materials List: paper, pencil, Exploring Exponents BLM

In this activity, students will work with a partner to discover the laws of exponents. Provide students with the Exploring Exponents BLM. Have them complete the chart and develop a formula for each situation. In the last column, students should write a verbal explanation of the rule that was discovered.

Discuss with students the formulas that they discovered and the explanations they wrote. Emphasize the concept of negative exponents as they were introduced in Unit 1.

Have students use *split-page notetaking* (view literacy strategy descriptions) to reinforce the rules of exponents. A sample of *split-page notetaking* is shown below.

A product of powers: $x^m \cdot x^n$	x^{m+n} When multiplying like bases, add the exponents
A quotient of powers: $\frac{x^m}{x^n}$	x^{m-n} When dividing like bases, subtract the exponents
A power to a power: $\left(x^m\right)^n$	x^{mn} When raising a power to a power, multiply the exponents
A fractional power raised to a power: $\left(x^{\frac{1}{n}}\right)^n$	$x^{\frac{n}{n}} = x$ When raising power with a unit fraction as the exponent to a power equal to the denominator, the base is raised to the first power.

Emphasize to students the importance of the final column as a means for later recall and application. Students can study from the *split-page notes* by covering one column and using the information in the other to try to recall the covered information. Students should also be allowed to quiz each other over the content of their notes.

Using a math textbook as a reference, provide examples and practice problems for students to simplify that include using order of operations.

Activity 7: Operations on Polynomials using Algebra Tiles (GLEs: <u>2</u>, 8: CCSS: <u>A-APR.1</u>)

Materials List: paper, pencil, Algebra Tile Template BLM

Give students examples of expressions that are and are not polynomials and help them to develop the definition of polynomial. Also, include an introduction on monomials, binomials, and trinomials. Allow students time to revisit their *vocabulary self-awareness charts* from Activity 1 to edit previous entries.

Divide students into groups and provide each group with a set of algebra tiles. Algebra tiles are manipulatives that help students visualize polynomial expressions. They can be made by using card stock and the Algebra Tile Template BLM. Use two different colors of card stock, one color to represent positive and the second color for negative values. Introduce algebra tiles to students and help them to understand the representation of each ($x^2, -x^2, x, -x, 1, -1$). Give students different polynomials, such as $2x^2 + 3x - 4$, and have the students model each polynomial with their algebra tiles. Discuss adding polynomials giving examples and have the students model each. Relate the process to combining like terms with which students have had experience in their work with solving equations. Also, emphasize that the set of polynomials is closed over addition because the set of real number coefficients is closed. Include a discussion of positive and negative tiles "canceling" out or adding up to zero.

Subtraction can be demonstrated by adding the opposite or changing the sign to addition and flipping the tiles in the expression's being subtracted. Continue emphasizing the closure property of polynomials over subtraction.

Multiplication of polynomials can be shown with algebra tiles by thinking of the two expressions being multiplied as the dimensions of a rectangle. The simplified expression is the area of the rectangle. Include examples of multiplying a monomial and a binomial and multiplying two binomials together. Repeat the reference to closure property as the discussion proceeds. Provide examples for groups to practice. Help students make the connection from concrete examples to abstract examples.

Using a math textbook as a reference, provide opportunities for students to practice simplifying polynomial expressions.

After students have had time to practice simplifying polynomial expressions, present them with the following problem:

> Farmer Ted wants to build a pen for his pigs. He is not sure how much fencing he needs, but he knows that he wants the length to be four feet more than the width. Write expressions for the length of the fencing that he needs and the area of the pig pen. *($(2x + 2(x + 4)) = 4x + 8$; $x(x + 4) = x^2 + 4x$)*

Once students have had time to complete the problem, model its solution. Sketch a rectangle, labeling the width as x and the length (x + 4). Remind students of the perimeter of a rectangle formula (P = 2(width) + 2(length)) then substitute into the formula: x in place width and (x + 4) in place of length. Therefore, $P = 2x + 2(x + 4)$. Using the distributive property results in the expression $2x + 2x + 8$; combining similar terms provides that the perimeter is represented by the expression: $4x + 8$. Using the area formula A = length(width), substitute, respectively, $(x + 4)$ for length and x for width. Emphasize the use of the distributive property: $A = x\,(x + 4)$. Remind students that $x(x)$ results in x^2 by adding the exponents and $4(x)$ is $4x$. Since there are no like terms, the expression remains $x^2 + 4x$.

Have students participate in a math *text chain* (view literacy strategy descriptions) activity to create word problems using polynomial expressions to solve geometric problems. After using the algebra tiles and seeing the above problem modeled, students should have a visual understanding of how polynomials could be used to solve word problems. First, form groups of four students. Ask the first student to initiate the story and pass the paper to the next student who adds a second line. The next student adds a third line, and the last student solves the problem. All group members should be prepared to revise the story based on the last student's input as to whether it was clear.

A sample *text chain* could be:

Student 1: Susie wants to put carpet in her bedroom.
Student 2: Her room is 3 feet longer than it is wide.
Student 3: Write an expression for the amount of carpet that she will need.
Student 4: $x(x + 3) = x^2 + 3x$

Have the different groups share their *text chains* with the rest of the class. Check problems for logic and correct solutions.

Activity 8: Arithmetic and Geometric Sequences (CCSS: F-LE.2)

Materials List: paper, pencil, calculator, Arithmetic and Geometric Sequences BLM

Students have already practiced with linear and exponential functions. After they have worked with both, students will need to recognize the algebraic differences between the arithmetic and geometric sequences. Refer to the *vocabulary self-awareness chart* to stress to students that geometric sequences are based on repeated common ratios, and arithmetic sequences are based on common differences. Provide the students with the formulas for arithmetic sequence

$A(n)$	$= a$	$+$	$(n - 1)$	(d)
↓	↓		↓	↓
*n*th term	1st term		term number	common difference

and for the geometric sequence: $A(n)$ = a · r^{n-1})

↓	↓	↓	↘
*n*th Term	1st term	common ratio	term number

Have students complete the *process guide* (view literacy strategy descriptions) Arithmetic and Geometric Sequences BLM. A *process guide* will scaffold students' comprehension within a unique format. The *process guide* is designed to stimulate students' thinking during or after reading and learning. The guide will also enable the students to focus on important information, in this case, the difference between an arithmetic and a geometric sequence. After students have completed the *process guide*, discuss students' answers and monitor their corrections. Have other students evaluate the examples made up by their classmates

<u>2013-2014</u>
Activity 9: Introducing Quadratic Equations (CCSS: <u>A-REI.4</u>)

Materials list: paper, pencil, Vocabulary Self-Awareness Chart 2 BLM

Vocabulary self-awareness charts (view literacy strategy descriptions) have been used often to build technical vocabulary fluency throughout the curriculum. A BLM is provided for students to use as they begin to study quadratic functions As students become proficient with the vocabulary, select student-made *vocabulary self-awareness charts* to place on the bulletin board to remind students about linear and exponential functions. *Vocabulary self-awareness charts* may also be used to review for tests and quizzes.

Once the *vocabulary self-awareness chart* has been completed, define the term quadratic equation for the students. Lead a discussion about how someone might try to solve a quadratic equation and how many solutions a quadratic equation might have. Then present students with a basic quadratic equation which requires students to take the square root of both sides: If $x^2 = a$, then x is the square root of a. (If $5^2 = 25$ then 5 is the square root of 25.) Emphasize that each positive number has both a positive and a negative square root. Hence, a quadratic of this form has two solutions. Therefore, in the example, the square root of 25 is ±5, with +5 being called the principal square root. Remind students that zero has one square root. Negative numbers have no real square roots, because the square of every real number is positive. Model several equations of the form $x^2 = a$ until you are satisfied with students proficiency in solving equations using square roots.

Extend the lesson to include quadratics of the standard form $ax^2 + bx + c = 0$, where $a \neq 0$. Start with equations in which $b = 0$: $ax^2 + c = 0$. To solve equations such as these, recall for students the need to isolate x^2 on one side, then find the square root(s) of both sides. Provide students with examples, such as $x^2 = 4$ ($x = \pm 2$); $x^2 = 5$ ($x = \pm\sqrt{5}$); $x^2 = 0$ ($x = 0$); $3x^2 - 16 = 32$ ($x = \pm 4$). Use the algebra text as a reference for more problems for students to complete.

2013-2014
Activity 10: Graphing Quadratic Equations (CCSS: A-REI.4, A-REI.10)

Materials List: paper, pencil, index cards, graph paper, Graphing Quadratic Equations BLM, graphing calculator (optional)

Begin graphing quadratic functions with an equation in the form $x^2 = y$. As each student enters the class, provide him/her with an index card with either a positive or a negative number. One student will receive 0. The closer the numbers are together, the better the graph will be (0.5 is a good interval between the values). Once the students have their index cards, have them square the values. On a coordinate plane on the board, have students plot the ordered pairs on the coordinate plane. If technology is available, use an interactive whiteboard to present the graph. The plotted points should be close enough together to simulate the smooth continuity of the curve of the parabola. Have students describe the graph by asking them questions such as where does the graph cross the *x*-axis? Have students predict what happens to the graph as *x* increases and as *x* decreases. Sketch a vertical line on the *y*-axis to highlight the axis of symmetry. Direct the students' attention to the point where the graph changes direction and the point where the graph crosses the *x*-axis. To further investigate quadratics and their parabolic graphs, use the interactive parabola tool at http://www.mathwarehouse.com/quadratic/parabola/interactive-parabola.php. The software will allow the students to discover what happens to a graph when the coefficients change. It also allows the student to look at graphs which have no real solutions, one solution or two solutions. Provide students with specific values of *a*, *b*, and *c* to use in the standard form $ax^2 + bx + c = y$ to enable a discussion regarding the graph.

Have students complete the Graphing Quadratics Equations BLM. Divide students into groups to complete each quadratic graph. Have as many examples as needed to give each group one problem. Have each group complete one example, determining graphically whether the quadratic has solutions. After each group completes the graph and each member of the group understands their example, use a modified form of *discussion* (view literacy strategy descriptions) to regroup the students based on the equation graphed so that each group has one member from every other group. Each member explains his/her problem to all other members of his new group. This strategy provides opportunities for students to demonstrate understanding of the newly-learned content regarding quadratics. In addition students have an immediate opportunity to share and apply the new information. Through *discussion*, students will develop a deeper understanding of quadratics. Monitor each group to ensure that all students understand all of the graphs and their solutions. Provide opportunities for students to discuss their observations about the solutions to each equation and the *x*-intercepts on the graph. Be sure students make the connection that the *x*-intercepts have the same *x*-value as the solution to the equations.

2013-2014

Activity 11: Solving Quadratic Equations by Factoring (CCSS: A-REI.4, A-REI.10)

Materials list: paper, pencil, graphing calculator (optional)

Factoring quadratics is another method that can be used to solve quadratic equations. Provide students with examples of graphs of quadratic functions that have real integral solutions. Ask students to recall that the *x*-intercepts of the parabolas are the solutions to the equations (refer to Activity 10). Provide students with examples of quadratics in factored form $(x-a)(x-b)=0$ and have students solve the equation using the zero-product property to find the *x*-intercepts. Have students factor quadratics in the form $ax^2+bx+c=0$, where $a = 1$. Also, provide examples in the forms of perfect square trinomials, differences of squares, and $ax^2+bx+c=0$, where $a > 1$. In addition, provide examples that cannot be factored or that have no real solution. Monitor progress and continue to offer examples as required by student skills.

2013-2014

Activity 12: Completing the Square and the Quadratic Formula (CCSS: A-REI.4, A-REI.10)

Materials List: paper, pencil, graphing calculator

Introduce completing the square as a method for factoring and solving quadratic equations. Review formulas for finding the vertex of a parabola from Activity 9. Use a numeric example to introduce the procedure. Call student attention to the process of completing the square for a quadratic: x^2+bx by adding the square of half the coefficient of *x*.

Example: $x^2+10x-24=y$

$x^2+10x-24=y$	Write the original equation.
$x^2+10x-24=0$	Replace the y with 0.
$x^2+10x=24$	Move the constant to combine it with 0.
$x^2+10x+5^2=24+5^2$	Add the square of half of the *b* to both sides of the equation.
$(x+5)^2=49$	Write the trinomial as a squared binomial. Simplify the right side.
$x+5=\pm 7$	Find the square root of both sides of the equation.
$x=-5\pm 7$	Subtract 5 from both sides of the equation.
$x=2$ or $x=-12$	Simplify as two expressions.

Then, show how a proof of the quadratic formula is developed. The quadratic formula is the method to determine all solutions, real and complex.

Use the following process of completing the square to develop the quadratic formula.

$$ax^2+bx+c=0$$
$$ax^2+bx=-c$$
$$x^2+\frac{b}{a}x=-\frac{c}{a}$$
$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{4ac}{4a^2}$$
$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$
$$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\sqrt{\frac{b^2-4ac}{4a^2}}$$
$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ provides the values of the roots, the number of solutions, and type of solution. Tell students that the discriminant $\left(b^2-4ac\right)$ determines the number and types of roots.

1. If $b^2-4ac=0\Rightarrow$ one zero and one real double root
2. If $b^2-4ac>0\Rightarrow$ two zeroes and two real roots
3. If $b^2-4ac<0\Rightarrow$ no zeroes and two imaginary roots

Emphasize the difference in the word *root*, which can be real or imaginary, and the word *zero*, which refers to an *x*-intercept of a coordinate graph.

Use a textbook as reference to assign problems regarding solving quadratics using the formula and to determine the discriminant of quadratics.

Sample Assessments

General Assessments

Performance and other types of assessments can be used to ascertain student achievement. Here are some examples.

- In Unit 1, the student compared two data sets of salaries as examples of linear and non-linear data. The students will revisit that report and find the regression equations for each set of data. The student will also make predictions using the equations.
- The student will obtain population data for Louisiana as far back as possible. The student will graph the data and find the regression equation. The student will then predict the population in the state for the year 2015. He/she will also compare the actual 2010 Louisiana population to an interpolated value from his/her exponential regression. The student will write a report summarizing his/her findings and include why it would be important to be able to estimate the future population of the state. The discussion should include reasons why (or why not) an exponential function would accurately reflect the state's population growth.
- The student will solve constructed response items such as this:
    - Over a one-year time period, an insect population is known to quadruple. The starting population is fifteen insects.
        a. Make a table and a graph to show the growth of the population from 0 through 6 years.
        b. How many insects would there be at the end of 10 years? (*15,728,640)*
        c. Write an exponential equation that describes the growth. ($y = 15 \cdot 4^x$)
        d. Would your equation correctly describe the insect population after 50 years? Justify your answer.
- The student will solve open response items such as:
    - Decide if the following situations are linear or exponential. Use examples to justify your answer.
        a. A constant change in the independent variable produces a constant change in the dependent variable. (*linear)*
        b. A constant change in the independent variable produces a constant percentage change in the dependent variable. (*Nonlinear)*
        c. A constant change in the independent variable produces two non-constant square ratios in the dependent variable. (*Quadratic)*
    - The perimeter of a rectangular concrete slab is 114 feet and its area is 702 square feet. Find the dimensions of the rectangle.
        a. Using l for the length of the rectangle, write an expression for the width of the rectangle in terms of l. ($57 - l = w$)
        b. Write a quadratic equation using l, the expression you found in part (a), and the area of the slab. *(* $l^2 - 57l + 702 = 0$ *)*
        c. Solve the quadratic equation. Use the two solutions to find the dimensions of the rectangle. *(The dimensions of the rectangle are 18 and 39 feet.)*

- The student will complete entries in his/her math *learning logs* using such topics as these:
  - Compare the graphs of $y = 4^x$ and $y = \left(\frac{1}{4}\right)^x$. How are they alike? How are they different?
  - Explain what is meant by exponential growth and exponential decay.
  - How many ways are there to write x^{12} as a product of two powers. Explain your reasoning.
  - Raul and Luther used different methods to simplify $\left(\frac{m^8}{m^2}\right)^3$. Are both methods correct? Explain your answer

    Raul

    $\left(\frac{m^8}{m^2}\right)^3 = \frac{m^{24}}{m^6} = m^{18}$

    Luther

    $\left(\frac{m^8}{m^2}\right)^3 = \left(m^6\right)^3 = m^{18}$
    - Describe some real-life examples of exponential growth and decay. Sketch the graph of one of these examples and describe what it shows.
    - Explain what is meant by a quadratic function.
    - What is the purpose of the discriminant of the quadratic formula?
    - On the graph of a quadratic, what is the significance of the *x*-intercept(s)?

Activity-Specific Assessments

- Activity 1:
  - Given an algebraic representation and a table of values of an exponential function, the student will verify the correctness of the values.
  - The student will demonstrate the connection between:
    - a constant rate of change and a linear graph
    - a varying rate of change and a nonlinear graph

- Activity 2: The student will decide which job offer he/she would take given the following two scenarios.
  Job A: A starting salary of \$24,000 with a 4% raise each year for ten years.
  Job B: A starting salary of \$24,000 with a \$1000 raise each year for ten years.
  The student will justify their answer with tables, graphs and formulas.

- Activity 3: The student will solve constructed-response items such as:
  Use the following data:

African Black Rhino Population

Year	Population (in 1000s)
1960	100
1980	15
1991	3.5
1992	2.4

a. Using your calculator and graphing paper, make a scatter plot of the data
b. Find the regression equation for the data. ($y = 1.74 \cdot 0.89^x$)
c. Use your model to predict the rhino population for the years 1998 and 2004. (*1,500; 770)*
d. Use your model to determine the rhino population in 1950. (*342,000)*
e. Should scientists be concerned about this decrease in population?
f. Compare your equation for M&M data to your equation for the rhino data. How are they alike? How are they different?

- Activity 4: The student will solve constructed-response items such as this:

The following data represents the number of people at South High who have heard a rumor:

# of hours after the rumor began	# of people who have heard it
0	5
1	10
2	20
3	40
4	80

a. Graph the data.
b. Find the exponential equation that models the data ($y = 5 \cdot 2^x$)
c. Use your equation to determine the number of people who have heard the rumor in 10 hours. (*5,120)*

- Activity 7: The student will simplify polynomial expression by addition, subtraction, and multiplication.

- Activity 10: The student will graph quadratic equations, find the *x*-intercepts, find the equations for the axes of symmetry, and find the vertices.

- Activity 12: The student will solve quadratic equations by completing the square and by using the quadratic formula.